



Water Supply and Sewerage Network
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Analysis Methods for Water Distribution Systems

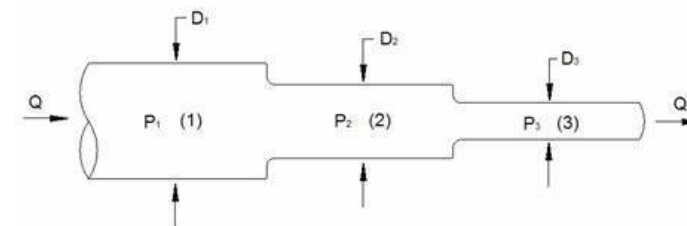
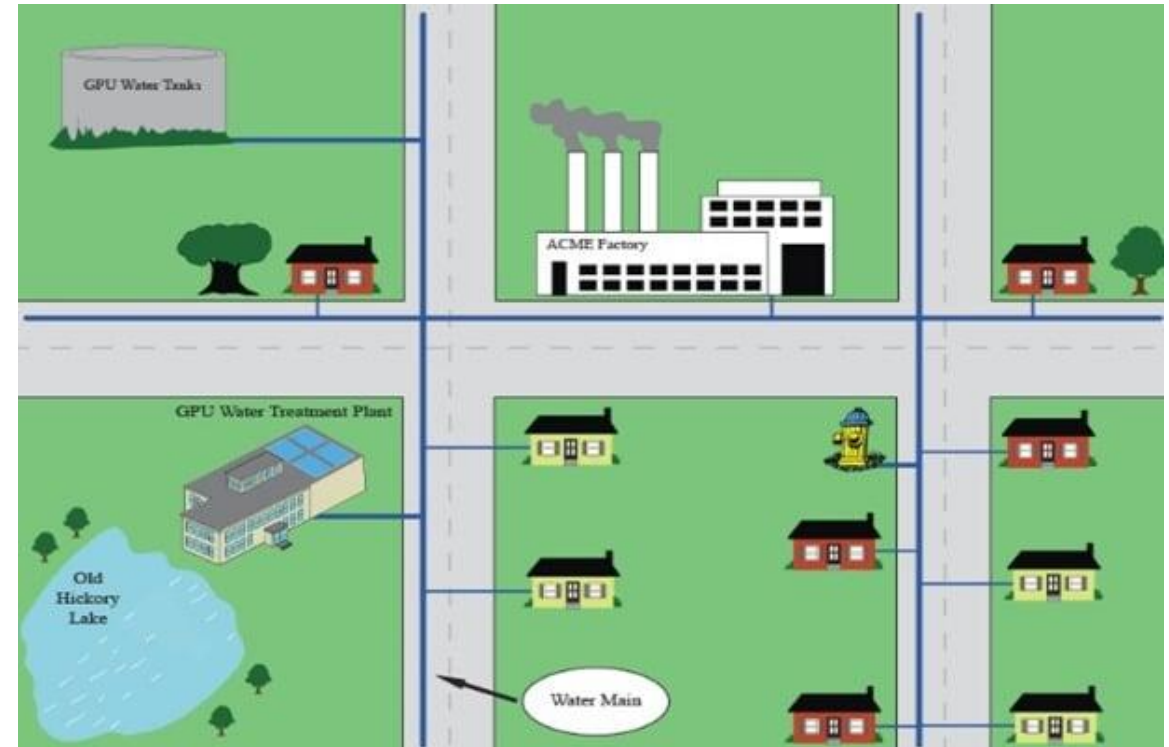
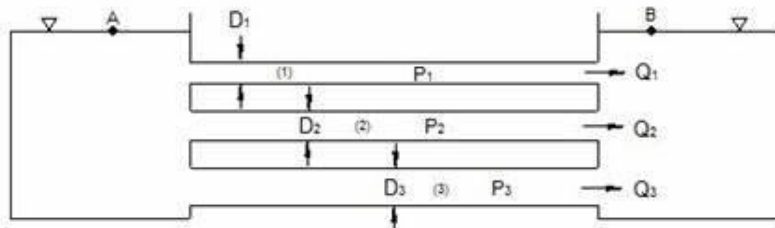
The purpose of these methods is to find the discharge for each pipe & the pressure at each junction (node)

❑ Method of Equivalent Pipes

It is used for changing complex pipes system to single equivalent line. This method cannot be applied directly to pipe systems containing crossovers or take-offs.

Principles:

1. Head losses through pipes in series are additive, while the discharge is identical.
2. Head losses through pipes in parallel are identical, while the discharge is additive.



Example:

Find an equivalent pipe for the network of Figure below. Express Q in mgd, S in %, H in ft. Use $C = 100$, & $Q = 1.5$ mgd.

Solution:

What are the required parameters for each pipe?

1. Line ABD

2 pipes in series (AB & BD)

a) Pipe AB Assume $Q = 1$ mgd. $L = 3000$ ft
 $Q = 1$ mgd & Dia. = 12in \rightarrow Diagram $\rightarrow S = 0.0021$
 $H_L = S \times L = 0.0021 \times 3000 = 6.3$ ft.

b) Pipe BD Assume $Q = 1$ mgd. $L = 4000$ ft
 $Q = 1$ mgd & Dia. = 16in \rightarrow Diagram $\rightarrow S = 0.00052$
 $H_L = S \times L = 0.00052 \times 4000 = 2.1$ ft.

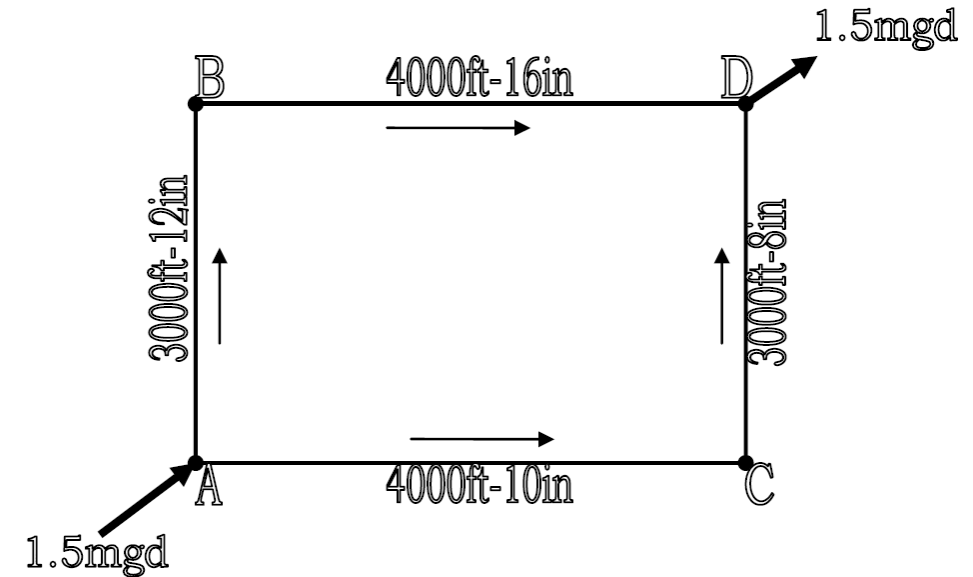
c) Total head losses $H_{ABD} = H_{AB} + H_{BD} = 6.3 + 2.1 = 8.4$ ft.

d) Line ABD, $Q = 1$ mgd & $H_L = 8.4$ ft, $D = ?$, $L = ?$

Any pipe that will do this is an equivalent pipe for line ABD. Choose 12in or 16in.

Now we choose 12in, $S = 0.0021$, $Q = 1$ mgd. (This need to modify its length)

Equivalent pipe for ABD: 12in, $S = 0.0021$, $L = 4000$ ft, $Q = 1$ mgd, Total $H_L = 8.4$ ft.



H_L , ft	Length, ft
2.1	1000
8.4	x

$$x = (8.4 \times 1000) / 2.1 = 4000 \text{ ft.}$$

2. Line ACD

2 pipes in series (AC & CD)

a) Pipe AC Assume $Q = 0.5\text{mgd}$, $L = 4000\text{ft}$
 $Q = 0.5\text{mgd}$ & Dia. = 10in \rightarrow Diagram $\rightarrow S = 0.00142$
 $H_L = S \times L = 0.00142 \times 4000 = 5.7\text{ft}$.

b) Pipe CD Assume $Q = 0.5\text{mgd}$, $L = 3000\text{ft}$
 $Q = 0.5\text{mgd}$ & Dia. = 8in \rightarrow Diagram $\rightarrow S = 0.0042$
 $H_L = S \times L = 0.0042 \times 3000 = 12.6\text{ft}$.

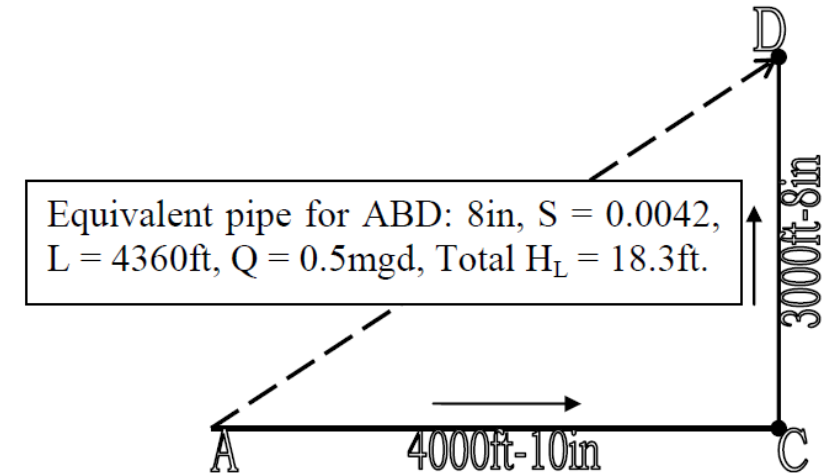
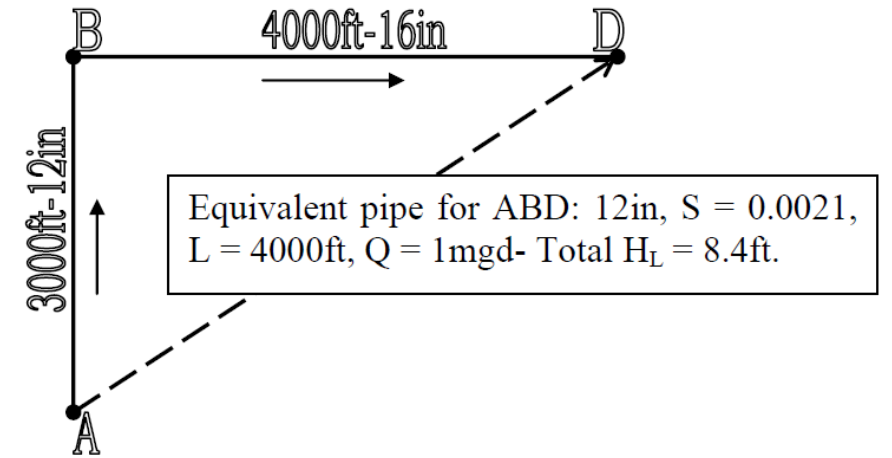
c) Total head losses $H_{ACD} = H_{AC} + H_{CD} = 5.7 + 12.6 = 18.3\text{ft}$.

d) Line ACD, $Q = 0.5\text{mgd}$ & $H_L = 18.3\text{ft}$, $D = ?$, $L = ?$
 Any pipe that will do this is an equivalent pipe for line ABD. Choose 10in or 8in.
 Now we choose 8in, $S = 0.0042$, $Q = 0.5\text{mgd}$. (This need to modify its length)

H_L , ft	Length, ft
4.2	1000
18.3	x

$$x = (18.3 \times 1000) / 4.2 = 4360\text{ft}$$

Equivalent pipe for ABD: 8in, $S = 0.0042$, $L = 4360\text{ft}$, $Q = 0.5\text{mgd}$, Total $H_L = 18.3\text{ft}$.



Important Note: Water always moves towards the least energy loss

3. Equivalent line AD

$H_L = 8.4\text{ft} = H_{ABD} = H_{ACD}$. What are the required parameters? $D=?$ $L=?$

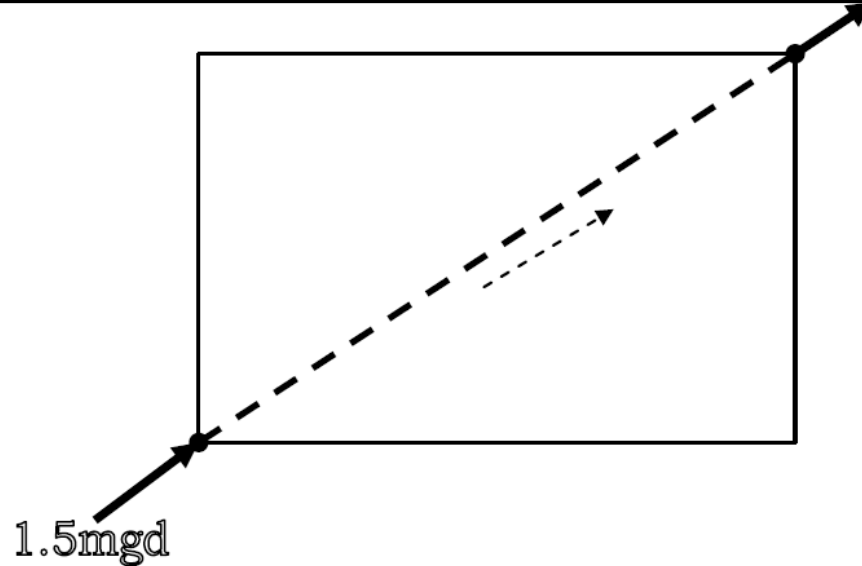
ABD & ACD in parallel with a given H , $Q = Q_{ABD} + Q_{ACD}$

$$Q = 1 + 0.5 = 1.5 \text{ mgd}$$

Use 12in or 8in? We use the bigger diameter or the next bigger diameter.... 16in.

Use 16in & $Q = 1.5\text{mgd}$. Diag. $\rightarrow S = 0.0014$ $S = H_L/L \rightarrow L = H_L/S = 8.4 / 0.0014 \rightarrow L = 6000 \text{ ft}$

Equivalent pipe for AD: 16in, $S = 0.0014$, $L = 6000\text{ft}$, $Q = 1.5\text{mgd}$, Total $H_L = 8.4\text{ft}$.



❑ Hardy Cross Method

The Hardy Cross method is an **iterative method for determining the flow** in pipe network systems where the inputs and outputs are known, but the flow inside the network is unknown. The solution can be done by a trial & error hand computation. Now the solution made by computers.

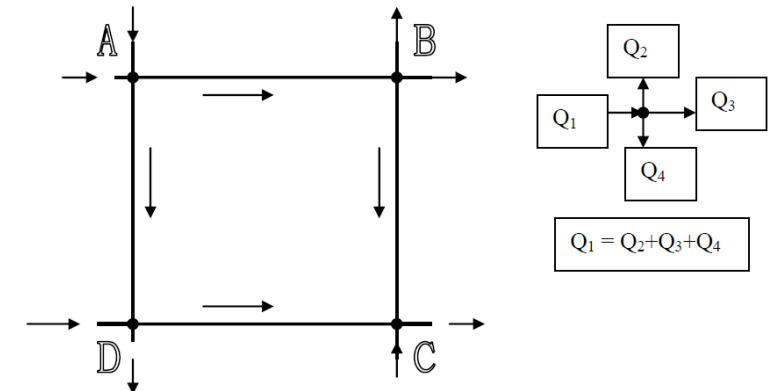
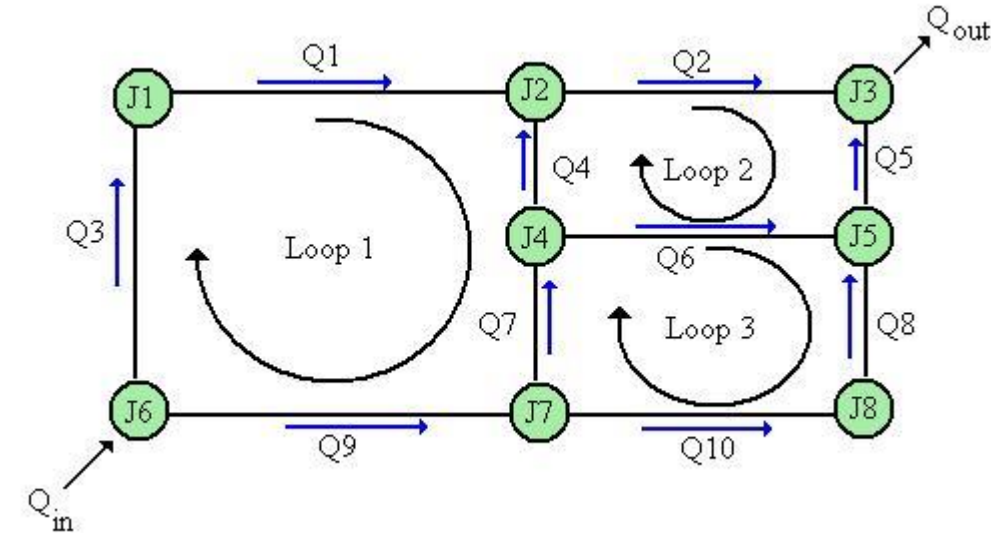
Basic Requirements:

1. Satisfy continuity, flow into & out each junction must be equal.
2. The head loss between any two junctions must be same.
3. The flow & head loss must be related by velocity-head loss equation.

➤ The general relationship between head loss and flow is:

$$h_f = KQ^n$$

Head loss equation	Relation	k	n
Hazen-Williams equation	$h_f = L \cdot \frac{10.67}{C^{1.85}} \frac{Q^{1.85}}{d^{4.87}}$	$L \cdot \frac{10.67}{C^{1.85}} \frac{1}{d^{4.87}}$	1.85
Darcy-Weisbach equation	$h_f = \frac{8fLQ^2}{g\pi^2 d^5}$	$L \cdot \frac{8f}{g\pi^2 d^5}$	2



Example: For the given source & loads shown in Figure, how will the flow be distributed in the simple network, and what will be the pressures at the load points if the pressure at the source is 60 psi? Assume horizontal pipes & $f=0.012$ for all pipes. Diameter & length of each pipe is indicated in the Figure.

Solution:

- Calculate head loss, K value for each pipe in the network using the following equation,

$$K = \frac{8 f L}{g D^5 \pi^2}$$

For pipe (1000ft, & 24in):
$$K = \frac{8 * 0.012 * 1000}{32.2 * \left(\frac{24}{12}\right)^5 * \pi^2} = 0.00944$$

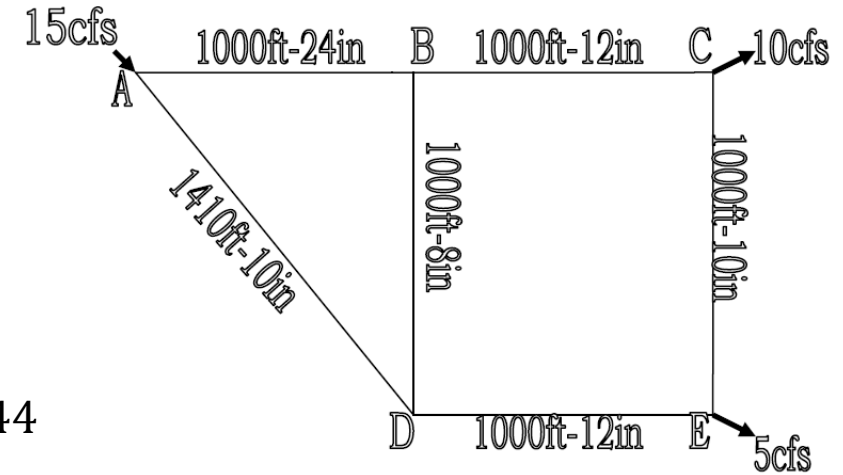
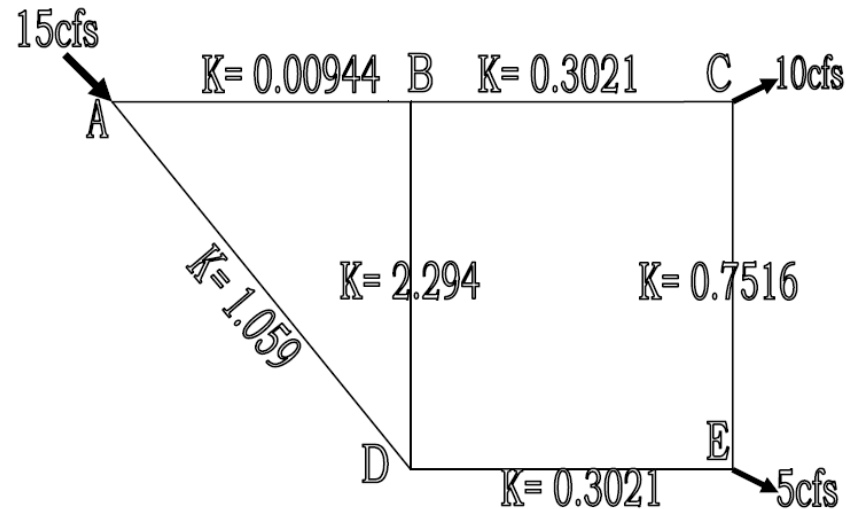


Figure below shows the network with the head loss, K value for each pipe.



- Distribute the discharge in the pipes and should satisfy that flow in and out from each junction is equal. Figure below shows the assumed flow in each pipe in the water network.

- ✓ The flow in the clockwise direction have positive (+ve) sign
- ✓ The flow in the counterclockwise have negative (-ve) sign

- Then we make the following table for each loop:

Pipe	K	Q	$h_f = KQ Q $	$2K Q $
AB	0.00944	10	0.944	0.1888
AD	1.059	-5	-26.475	10.59
BD	2.294	0	0	0

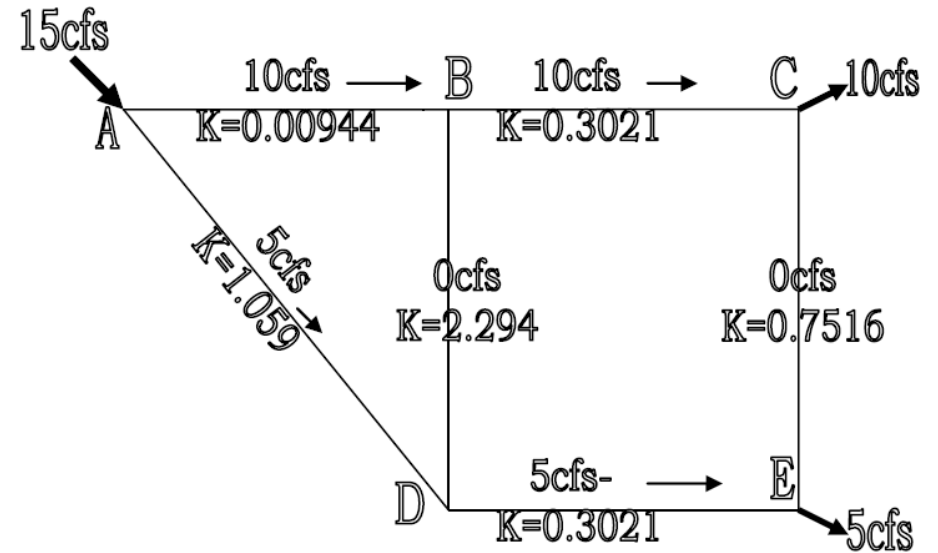
Σ	-25.531	10.7788
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$\Delta Q =$	-2.4
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Pipe	K	Q	$h_f = KQ Q $	$2K Q $
BC	0.3021	10	30.21	6.042
BD	2.294	0	0	0
CE	0.7516	0	0	0
DE	0.3021	-5	-7.5525	3.021

Σ	22.6575	9.063
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$\Delta Q =$	2.5
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The New Flow for 2nd Iteration

Loop 1			
Pipe	AB	AD	BD
Q	12.4	-2.6	4.9

Loop 2				
Pipe	BC	BD	CE	DE
Q	7.5	-4.9	-2.5	-7.5

- To finish the 1st iteration, the corrections obtained in the table are applied to the two loops, and the pipe discharges are shown in Figure below

✓ The New Flow = Old Flow - ΔQ

✓ The New Flow (Shared Pipe)= Old Flow - ΔQ (I) + ΔQ (II)

The 2nd Iteration

Pipe	K	Q	$h_f=KQ Q $	$2K Q $
AB	0.00944	12.4	1.451494	0.234112
AD	1.059	-2.6	-7.15884	5.5068
BD	2.294	4.9	55.07894	22.4812

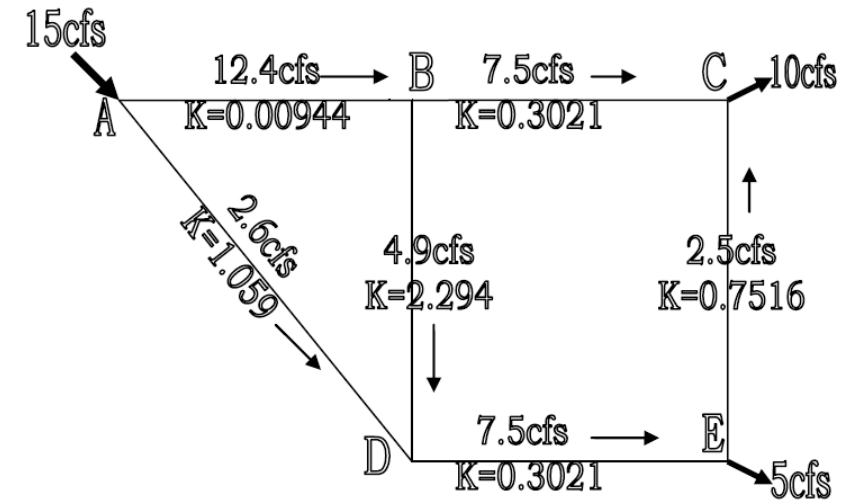
Σ	49.37159	28.22211
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$\Delta Q =$	1.7
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Pipe	K	Q	$h_f=KQ Q $	$2K Q $
BC	0.3021	7.5	16.99313	4.5315
BD	2.294	-4.9	-55.0789	22.4812
CE	0.7516	-2.5	-4.6975	3.758
DE	0.3021	-7.5	-16.9931	4.5315

Σ	-59.7764	35.3022
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$\Delta Q =$	-1.7
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The New Flow for 3rd Iteration

Loop 1			
Pipe	AB	AD	BD
Q	10.7	-4.3	1.5

Loop 2				
Pipe	BC	BD	CE	DE
Q	9.2	-1.5	-0.8	-5.8

The 3rd Iteration

Pipe	K	Q	$h_f = KQ Q $	$2K Q $
AB	0.00944	10.7	1.080786	0.202016
AD	1.059	-4.3	-19.5809	9.1074
BD	2.294	1.5	5.1615	6.882

Σ	-13.3386	16.19142
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$\Delta Q =$	-0.8
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Pipe	K	Q	$h_f = KQ Q $	$2K Q $
BC	0.3021	9.2	25.56974	5.55864
BD	2.294	-1.5	-5.1615	6.882
CE	0.7516	-0.8	-0.48102	1.20256
DE	0.3021	-5.8	-10.1626	3.50436

Σ	9.764576	17.14756
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$\Delta Q =$	0.6
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The New Flow for 4th Iteration

Loop 1			
Pipe	AB	AD	BD
Q	11.5	-3.5	2.9

Loop 2				
Pipe	BC	BD	CE	DE
Q	8.6	-2.9	-1.4	-6.4

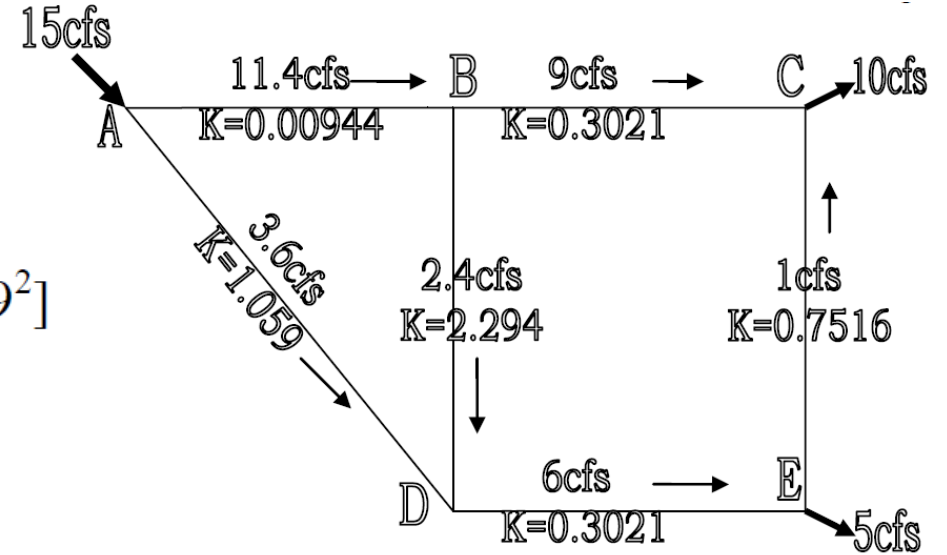
✓ Important Note: We have to keep trying the iterations until we reach the satisfied ΔQ

The final distribution of flow is obtained as shown in Figure

The pressures at the load points (C & E) are calculated as follows

$$\begin{aligned}
 P_C &= P_A - \gamma(K_{AB}Q_{AB}^2 + K_{BC}Q_{BC}^2) \\
 &= 60\text{psi} \times (144\text{psf/psi}) - 62.4[0.00944 \times (11.4)^2 + 0.3021 \times 9^2] \\
 &= 8640\text{psf} - 1603\text{psf} \\
 &= 7037\text{psf} \\
 &= \underline{48.9\text{psi}}
 \end{aligned}$$

$$\begin{aligned}
 P_E &= 8640 - \gamma(K_{AD}Q_{AD}^2 + K_{DE}Q_{DE}^2) \\
 &= 8640 - 62.4[1.059 \times (3.5)^2 + 0.3021 \times 6^2] \\
 &= 7105\text{psf} \\
 &= \underline{49.3\text{psi}}.
 \end{aligned}$$





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